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Fresh Look and Understanding on Carnot Cycle

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Abstract

Carnot cycle and theorem are major contributions of classical thermodynamics since proposed in 1824, while Carnot failed to prove it because of the influence of Caloric theory of heat. Then in 1850, Clausius confirmed Carnot theorem via reduction to absurdity and put forward the second law of thermodynamics. From then on, the theory that Carnot cycle is the one with the highest efficiency is generally accepted by scholars and regarded as the key premise or basis of thermodynamic analysis. However, few researchers care about how Carnot proposed the theory with the absence of the second law of thermodynamics. Can we get and prove Carnot theorem without the help of the second law of thermodynamics? In this paper, based on the polytropic process equations and p - v diagram, an analytical method of thermodynamics is developed and the cycle with the highest efficiency is deduced out only through extremum principle, instead of the second law of thermodynamics. The preliminary results indicate that Carnot cycle just serves as one form of the best cycles and only if the polytropic exponents (n) of the expansion and compression processes equal to each other, the efficiency of that cycle reaches the highest value, just that of Carnot cycle. This work gives a new insight into Carnot theorem and classical thermodynamics, which is of great importance in enlightenment for applying the new analysis method to address the complicated problems which are very difficult to address by using the second law of thermodynamics.

Keywords: Carnot cycle; Engine efficiency; Extremum principle; Thermodynamics

1. Introduction

About half a century after James Watt presented the steam engine, Carnot proposed the theories including the Carnot cycle and the Carnot's theorem to illustrate what the highest efficiency is for a thermodynamic cycle. But deeply influenced by the Caloric theory, Carnot failed to prove his theorem. Until 1850, Clausius put forward the concept of entropy which constitutes the major foundation of the second law of thermodynamics and validated the Carnot's theorem perfectly via reduction to absurdity [1]. After that, the Carnot's theorem is always regarded as the origin of thermodynamic analysis while few people cares about how Carnot proposed his theorem with the absence of the second law of thermodynamics. Can we get and prove the Carnot's theorem without the second law of thermodynamics?

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Fig.1 shows a famous problem called “Brachistochrone” in mechanics theory [2]. Through classical mechanics, it is difficult to find the most time-saving path for an object sliding from point A to B only by gravity. However, through analytical mechanics, this problem can be addressed by variation extremum principle. This raises a further question on whether we can solve the aforementioned thermodynamic problem resorting to the extremum principle, since some similarities exist between them, in terms of there are heat source and sink with two certain temperatures in Carnot cycle, just like the points with two certain locations in “Brachistochrone”.

In this paper, resorting to extremum principle in analytical mechanics, we re-explore the efficiency of thermodynamic cycles from another perspective, to find out the highest efficiency of engine cycles. The objective is to make an attempt to find a new analysis method for thermodynamic optimizations. The preliminary results indicate that the best process can be deduced out only by using the first law and extremum principle, instead of the second law of thermodynamics. This work gives a new insight into engine cycles, which is of great significance in enlightenment for new analysis method to solve thermodynamic optimization problems.

2. Mathematical model

For a thermodynamic cycle, there are several processes. To simplified analysis, the cycles consisting of two isothermal processes and two uncertain ones are of only concern. It is assumed that there is no heat leakage or frictional loss for reversible cycles. For Carnot cycle, because only one heat source and one heat sink exist, the whole process is unique:

- 1-2:reversible isothermal expansion of the gas at the “hot” temperature, T_1
- 2-3: isentropic (reversible adiabatic) expansion of the gas (isentropic work output)
- 3-4:reversible isothermal compression of the gas at the “cold” temperature, T_2
- 4-1:isentropic compression of the gas (isentropic work input)

Then how about the efficiency of cycles where the steps 2-3 and 4-1 are not isentropic? If we take heat recovery into consideration, the cycle is totally possible. Hence, in engine cycles, the purpose is to find the best cycle form with the highest efficiency working under a heat source and sink of given temperatures.

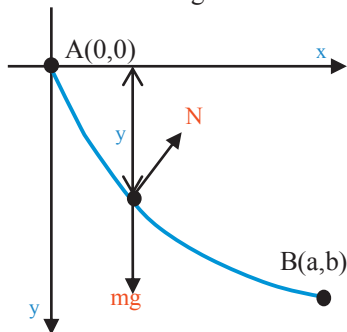


Fig.1. Brachistochrone curve in a coordinate system

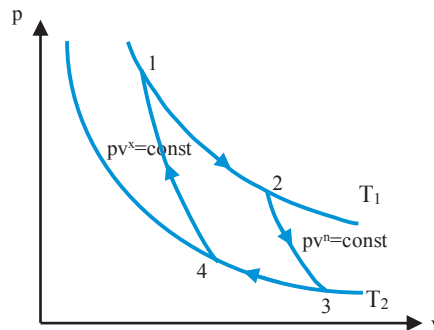


Fig. 2. A certain cycle illustrated on a p - v diagram

In the above cycle, the hot/cold temperature is T_1/T_2 , so steps 1-2 and 3-4 are unique (isothermal) while steps 2-3 and 4-1 are not. Actually the two processes' polytropic indexes may not be constant. Nonetheless, for an infinitesimal process, the polytropic index can be regarded as a constant value. Thus we define the polytropic index n and x for steps 2-3 and 4-1, respectively as showed in Fig.2. The heat transfer and work output/input for each step are Q_{1-2} , Q_{2-3} , Q_{3-4} , Q_{4-1} and W_{1-2} , W_{2-3} , W_{3-4} , W_{4-1} respectively. The engine

efficiency is expressed by Eq. (1). Then the purpose is to find out the relationship between n and x , when the engine efficiency arrives at maximal value.

$$\eta = \frac{RT_1 \ln \frac{v_2}{v_1} - RT_2 \ln \frac{v_4}{v_3} - \left\| \frac{R}{n-1} (T_1 - T_2) \right\| - \left\| \frac{R}{x-1} (T_2 - T_1) \right\|}{RT_1 \ln \frac{v_2}{v_1} + \left\| \left(c_v - \frac{R}{n-1} \right) (T_2 - T_1) \right\| - \left\| \left(c_v - \frac{R}{x-1} \right) (T_1 - T_2) \right\|} \quad (1)$$

Where η —cycle efficiency, R —universal gas constant, v_i —specific volume of state i

The engine efficiency must vary with different polytropic index x . And $y=x/n$ is designated as the evaluating criteria. In the following case study, we set $T_1=500$ K, $T_2=300$ K and the calculation results are shown in Fig.3.

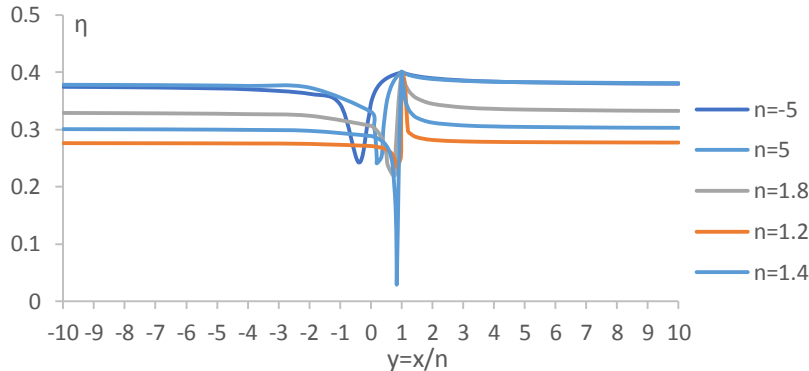


Fig. 3. Calculating result of efficiency with different y value

It can be seen from the curves that the optimal point is non-differential, so only numerical method should be used. The results indicate that when $y=1$ ($x=n$), the η reaches the maximum value, because of the total heat recovery between steps 2-3 and 4-1. So such situations, the thermodynamic cycle efficiency is same to the Carnot cycle (Eq. 2).

$$\eta = \frac{RT_1 \ln \frac{v_2}{v_1} - RT_2 \ln \frac{v_4}{v_3}}{RT_1 \ln \frac{v_2}{v_1}} = 1 - \frac{T_2}{T_1} \quad (2)$$

In summary, without the help of the second law of thermodynamics, the best cycle process with the highest efficiency can also be obtained by using extremum principle, just like “Brachistochrone” problem in analytical mechanics. So for two given heat source and sink temperatures, Carnot cycle serves as only one form of thermodynamic cycles with the highest efficiency. The aforementioned cycle is called the recapitulative Carnot cycle. The following section shows some examples where $x=n$.

3. Illustrative Example and Applications

From the aforesaid discussion, for given two heat source and sink temperatures, only if the expansion and compression processes share the same polytropic index, the engine efficiency always reaches the maximal value. For example, the processes of the Stirling cycle is quite similar to the Carnot cycle, and the difference is that the two isentropic processes are changed to isovolumetric ones. The efficiency of the Stirling cycle is same to the Carnot ones between the same heat source and sink. And the Ericsson cycle is the quite similar to the Stirling cycle except that the corresponding isovolumetric processes in the latter

become isobaric ones. So the Carnot cycle only serves as one form of the cycles between certain heat source/sink temperatures with highest efficiency [3].

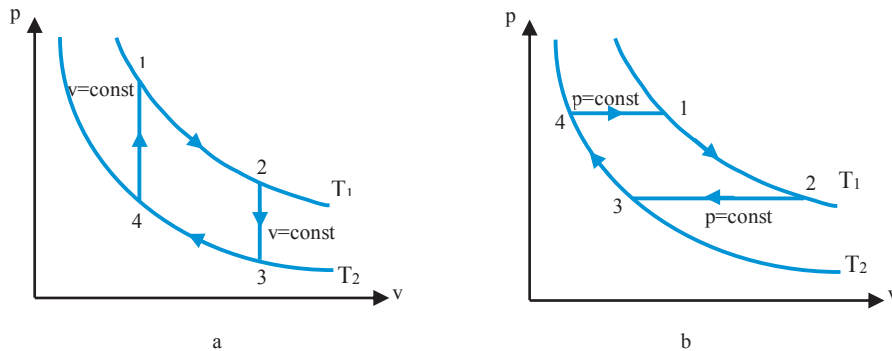


Fig. 4. The Stirling cycle (a) and the Ericsson cycle (b) illustrated on a p - v diagram

4. Conclusions

In this paper, a kind of analytical method to explore the efficiency of thermodynamic cycles is developed. And it can be applied to analyze thermodynamics processes. The results show that the engine cycle with highest efficiency can be deduced out by extremum principle, instead of the second law of thermodynamics. And only if the polytropic indexes of expansion and compression processes equal to each other, the engine efficiency always reaches the maximal value, just that of Carnot cycle. This work gives a new insight into engine cycles, which is of great significance in enlightenment for new analytical method to solve thermodynamic optimization problems. It also raises further question on whether we can find some examples where the new analysis method can address a practical problem while traditional thermodynamic method cannot. So this topic is really worth deep future investigations.

Acknowledgements

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References

- [1] Francesco di L. Complexity in the stepwise ideal gas Carnot cycle. *Physica A: Statistical Mechanics and its Applications*, Volume 314, Issues 1–4, 1 November 2002, Pages 331–344.
- [2] Golubev Y. F. Brachistochrone for a rigid body sliding down a curve. *International Journal of Computer and Systems Sciences*. 2013, 52(4): 571–587.
- [3] J. Castaing-Lasvignottes, P. Neveu. Equivalent Carnot cycle concept applied to a thermochemical solid/gas resorption system. *Applied Thermal Engineering*, Volume 18, Issues 9–10, 1 September 1998, Pages 745–754.



Biography

Rui Shang is a Master candidate in the department of building science, school of architecture, Tsinghua University. He got bachelor's degree of engineering in Tongji University in 2013. His research interest focuses on building energy efficiency.